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06MAT21

Second Semester B.E. Degree Examination, December 2011
Engineering Mathematics - II

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer FIVE full questions choosing at least two from each part.***
2. Answer all objective type questions only in OMR sheet page 5 of the Answer Booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Select the correct answer :

i) For the curve $r = a(a + \cos\theta)$, ρ^2 / r is
A) r B) $\theta^3/9$ C) 8a/9 D) $8/a^3$

ii) The value of c of the Rolle's theorem for $F'(x) = x^2 - 6x + 8$ in [2, 4] is
A) 3 B) – 3 C) – 2 D) – 1

iii) The Maclaurin's series expansion of cosx is
A) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots\ldots\ldots$ B) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots\ldots\ldots$
C) $1 + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\ldots$ D) None of these.

iv) The Lagrange's mean value theorem is a special case of
A) Rolle's theorem B) Cauchy's mean value theorem
C) Taylor's theorem D) Maclaurin's series. **(04 Marks)**

b. Derive an expression for radius of curvature in case of polar curves $r = f(\theta)$. **(06 Marks)**

c. Verify the Rolle's theorem for $f(x) = (x - a)^m (x - b)^n$ in [a, b]. Given m and n are +ve integers. **(04 Marks)**

d. Using the Maclaurin's series, prove that $\sqrt{1 + \sin 2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} - \ldots\ldots$ **(06 Marks)**

2 a. Select the correct answer :

i) The value of $\lim_{n \to \infty} (1 + x)^{1/x}$ is
A) e B) 1 C) 1/e D) ∞

ii) If $e^x \cos y = \frac{e}{\sqrt{2}}\left[1 + (x-1) - (y - \frac{\pi}{4}) + \frac{(x-1)^2}{2} - (x-1)(y - \frac{\pi}{4}) - \ldots\ldots\right]$ is the Taylor's expansion about the point
A) (0, 0) B) (1, 1) C) (1, $\pi/4$) D) ($\pi/4$, 1)

iii) If $rt - s^2 > 0$, $r < 0$ then f(a, b) is the
A) maximum value of f(x, y) B) minimum value of f(x, y)
C) saddle point D) None of these.

iv) The rectangular box of maximum volume and a given surface area is a
A) triangle B) rectangle C) cube D) None of these. **(04 Marks)**

b. Evaluate $\lim_{x \to 0}\left(\frac{a^x + b^x + c^x + d^x}{4}\right)^{1/x}$ **(04 Marks)**

c. Expand $\tan^{-1}(y/x)$ about the point (1, 1) upto the third degree term. **(06 Marks)**

d. The temperature T at any point (x, y, z) in space is $T = 40xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. **(06 Marks)**

3 a. Select the correct answer :

i) Value of $\int_1^2\int_1^3 xy^2 dxdy$ is

A) 10 B) 8 C) 13 D) - 13

ii) Value of $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)dxdydz$ is

A) 8π B) 1 C) $\frac{8}{3}abc(a^2+b^2+c^2)$ D) $\frac{3}{8}abc(a^2+b^2+c^2)$

iii) For $\int_0^\infty\int_x^\infty f(x,y)dxdy$, the change of order is

A) $\int_x^\infty\int_0^\infty f(x,y)dxdy$ B) $\int_0^\infty\int_y^\infty f(x,y)dxdy$ C) $\int_0^\infty\int_0^y f(x,y)dxdy$ D) $\int_0^\infty\int_0^x f(x,y)dxdy$

iv) The value of $\Gamma(n+1)$ is

A) 2 B) n + 1 C) (n + 1)! D) n! **(04 Marks)**

b. Evaluate $\int_0^\infty\int_0^{\sqrt{1-y^2}} x^3y\, dxdy$ **(04 Marks)**

c. Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}}\frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$ **(06 Marks)**

d. Prove that $\int_0^\infty\frac{dx}{1+x^4}=\frac{\pi}{2\sqrt{2}}$ **(06 Marks)**

4 a. Select the correct answer :

i) For a vector function $\overline{F}$, there exists a scalar potential, only when

A) div $\overline{F}=0$ B) grad(div $\overline{F}$) = 0 C) curl $\overline{F}=0$ D) $\overline{F}$ curl $\overline{F}=0$

ii) If the vector functions $\overline{F}$ and $\overline{G}$ are irrotational, then $\overline{F}\times\overline{G}$ is

A) irrotational B) solenoidal

C) both irrotational & solenoidal D) None of these.

iii) The Gauss divergence theorem is a relation between

A) a line integral and a surface integral B) a surface integral and a volume integral

C) a line integral and a volume integral D) two volume integrals

iv) A force field $\overline{F}$ is said to be conservative if

A) curl $\overline{F}=0$ B) grad $\overline{F}=0$ C) div $\overline{F}=0$ D) curl(grad $\overline{F}$) = 0 **(04 Marks)**

b. Use divergence theorem to evaluate $\int_s \overline{A}\cdot\hat{n}\,ds$, where $\overline{A}=x^3i + y^3j + z^3k$ and s is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **(04 Marks)**

c. Verify the Green's theorem for $\int_c (xy + y^2)dx + x^2 dy$, where c is bounded by y = x and $y = x^2$. **(06 Marks)**

d. Prove that $\overline{A} = \frac{\cos\theta}{r^3}\left[\frac{\hat{e}_r}{\sin\theta} + \frac{\hat{e}_\theta}{\cos\theta} + r^4 e_\phi\right]$ is solenoidal ($\overline{A}$ is spherical polar system.) **(06 Marks)**

PART – B

5 a. Select the correct answer :

i) The complementary function of $\frac{d^2y}{dx^2} + 4y = 5$ is

A) $c_1 \sin 2x + c_2 \sin 3x$ B) $c_1 \cos 2x + c_2 \sin 2x$
C) $c_1 \cos 2x - c_2 \sin 2x$ D) None of these.

ii) The particular integral of $(D^2 - 4)y = \sin 2x$ is

A) $\frac{x}{2}\sin 2x$ B) $\frac{-x}{4}\cos 2x$ C) $\frac{x}{2}\cos 2x$ D) None of these.

iii) The solution of the differential equation $(D^2 - 2D + 1)y = 0$ is

A) $c_1 e^x + c_2 e^{-x}$ B) $c_1 + c_2 x)e^x$ C) $c_1 e^{-x}$ D) $c_1 + c_2 e^{-2x}$

iv) The solution of a differential equation which is not obtained from the general solution is known as

A) Particular solution B) Singular solution
C) Complete solution D) Auxiliary solution. **(04 Marks)**

b. Solve $(D^4 + 4D^3 - 5D^2 - 36D - 36)y = 0$ **(04 Marks)**

c. Solve $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x} + \sin 2x$ **(06 Marks)**

d. Solve $(D^2 + 3D + 2)y = 1 + 3x + x^2$. **(06 Marks)**

6 a. Select the correct answer :

i) The Wronskian of cos2x and sin2x is

A) w = 4 B) w = 1 C) w = 2 D) w = 3/2

ii) To transform $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = \sin 2[\log(1+x)]$ into a linear differential equation with constant coefficient, we put 1 + x =

A) log x B) e^x C) e^z D) z.

iii) By the method of variation of parameters, the formula for A′ is

A) $\frac{y_1\phi(x)}{w}$ B) $\frac{y_2\phi(x)}{w}$ C) $\frac{-y_2\phi(x)}{w}$ D) None of these.

iv) The initial value of problem $\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 0$; x(0) = 0 is

A) $c_1 - c_2 = 0$ B) $c_1 + c_2 = 0$ C) $c_1 = 0$ D) $c_2 = 0$ **(04 Marks)**

b. Solve by the method of variation of parameters $\frac{d^2y}{dx^2} + 4y = \tan 2x$. **(04 Marks)**

c. Solve $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 9y = 3x^2 + \sin(3\log x)$ **(06 Marks)**

d. Solve $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 13x = 0$, with x(0) = 0 and dx(0)/dt = 2. **(06 Marks)**

7 a. Select the correct answer :

i) The Laplace transform of f(t), $t \geq 0$ is defined by

A) $\int_0^\infty e^{-st} f(t)dt$ B) $\int_0^\infty e^{st} f(t)dt$ C) $\int_t^\pi e^{-st} f(t)dt$ D) $\int_{-\infty}^\infty e^{-st} f(t)dt$

ii) The Laplace transform of cos2t is

A) $\frac{1}{s^2+2^2}$ B) $\frac{s}{s^2+2^2}$ C) $\frac{2}{s^2+2^2}$ D) $\frac{s^2}{s^2+2^2}$

iii) The Laplace transform of f(t)/t is

A) $\int_s^\infty F(s)ds$ B) $\int_0^\infty F(s)ds$ C) $\int_1^\infty \frac{1}{s} F(s)ds$ D) $\int_{-\infty}^\infty \frac{F(s)}{s} ds$

iv) The Laplace transform of $e^{(t-1)} H(t-1)$ is

A) $\frac{e^{-s}}{s-1}$ B) $\frac{e^{s}}{s+1}$ C) $\frac{1}{s+1}$ D) e^{-s} **(04 Marks)**

b. Find the Laplace transform of $e^{-3t} \sin 5t \sin 3t$ **(04 Marks)**

c. Find the Laplace transform of the full wave rectifier f(t) = E sinwt, where $0 < t < \pi/w$, having the period (π/w). **(06 Marks)**

d. If $f(t) = \begin{cases} t^2, & \text{if } 0 < t \leq 3 \\ 4, & \text{if } t > 3 \end{cases}$,

express the f(t) in terms of unit step function and hence find its Laplace transform. **(06 Marks)**

8 a. Select the correct answer :

i) The inverse Laplace transform of $\frac{s^3+s^2+6}{s^4}$ is

A) $1+t+t^3$ B) $2+3t+t^4$ C) $\frac{1+t^3}{t}$ D) $t+t^2+3t^3$

ii) The inverse Laplace transform of $\frac{s}{(s^4+a^2)^2}$ is

A) sinat – atcosat B) $\frac{1}{2a}\cos at$ C) $\frac{1}{2a} t \sin at$ D) t cos at

iii) The inverse Laplace transform of $\frac{s+b}{s+a}$ is

A) $\frac{1-e^{at}}{t}$ B) $\frac{e^{-at}-e^{-bt}}{t}$ C) $\frac{e^{at}+e^{bt}}{t}$ D) $\frac{1-\cos at}{t}$

iv) The inverse Laplace transform of unit step function H(t – a) is

A) e^{-at} B) F(s) C) $\frac{1}{s}e^{-as}$ D) $\frac{1}{s}$ **(04 Marks)**

b. Find the inverse Laplace transform of $\frac{2s-1}{s^2+2s+17}$. **(04 Marks)**

c. Find $L^{-1}\left[\frac{2s^2-6s+5}{s^3-6s^2+11s-6}\right]$ **(06 Marks)**

d. Using the Laplace transform technique, solve $\frac{d^2y}{dt^2}+4\frac{dy}{dt}+3y=e^{-t}$. Given $y(0)=y'(0)=1$. **(06 Marks)**

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